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Arch perimeter prediction using Ramanujan's equation for ellipse: A mathematical approach in borderline cases

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ABSTRACT

Objectives: The objective is to evaluate the accuracy of Ramanujan's equation for the perimeter of an ellipse in predicting pre- and post-treatment maxillary arch perimeter in borderline orthodontic cases.

Material and Methods: This retrospective cohort study analyzed 70 maxillary dental study models (35 pre-treatment and 35 post-treatment) from non-extraction orthodontic cases with borderline crowding or generalized spacing. The semi-major and semi-minor axes were measured using a digital Vernier caliper, and the arch perimeter was recorded manually using a brass wire. Ramanujan's equation for the perimeter of an ellipse was used to calculate theoretical arch perimeter values, which were then compared with measured values. Measurement reliability was evaluated using Cronbach's alpha. Validity was assessed through paired t-tests and intra-class correlation coefficients (ICC). The study was conducted following the strengthening of the reporting of observational studies in epidemiology guidelines.

Results: The mean difference between calculated and manual arch perimeter values was below 1.2% for both pre- and post-treatment models. ICC values were 0.985 (pre-treatment) and 0.977 (post-treatment), indicating excellent reliability. Paired *t*-tests showed no significant difference between manual and calculated values ($P > 0.05$). A strong linear correlation was confirmed through scatter plot analysis.

Conclusion: Ramanujan's equation demonstrated high reliability and predictive accuracy for estimating maxillary arch perimeter in borderline orthodontic cases. It may serve as a valuable mathematical adjunct in clinical decision-making, particularly in evaluating space availability for non-extraction treatment planning.

Keywords: Borderline cases, Ellipse, Mathematical model, Non-extraction treatment, Ramanujan's equation

INTRODUCTION

One of the most critical elements in orthodontic treatment planning is arch perimeter prediction, especially in non-extraction approaches. Traditional methods involve arch expansion or incisor proclination to address mild crowding or spacing.[1,2] The accurate estimation of space availability and the potential to gain space without extractions can significantly impact treatment decisions, particularly in borderline cases.

The studies by Bolton[3] suggested that a mathematical ratio is present between the maxillary and mandibular arches in ideal arch forms, and mathematical modeling of dental arches, particularly using Ramanujan's equation of the perimeter of an ellipse, offers a promising predictive tool.[4]

Several geometric models–including parabolas, catenary curves, and splines–have been proposed to represent dental arch forms.[5] Among these, the ellipse has shown a strong anatomical correlation with the natural curvature of the maxillary arch. Currier demonstrated that an elliptical model offers a better fit for dental arches compared to other geometric forms. However, despite this recognition, few studies have explored the clinical utility of elliptical equations in predicting actual arch perimeter values.[6]

A mathematical model of the mandibular arch, based on cubic spline interpolation,[7] was developed to quantitatively compare the effects of various types of orthodontic expansion on arch perimeter considering average arch dimensions such as intermolar width, intercanine width, and midline arch length–and was further supported by the geometric equation formulated by Mutinelli *et al*. to represent the mandibular arch form.[8]

The formulation of a suitable approximation to represent the ellipse has been attempted by several mathematicians. However, it is widely accepted that Srinivasan Ramanujan's equation, which he developed in 1914, is the most accurate.[9] This study builds on the foundation laid by previous researchers, especially Chung and Wolfgramm[4] and Antala *et al*.,[5] and aims to validate Ramanujan's equation using a new clinical dataset. This work not only evaluates the predictive accuracy of this equation but also explores its practical utility in orthodontic treatment planning.

However, existing literature is limited in its application of this model to post-treatment scenarios or in evaluating changes due to orthodontic interventions such as molar expansion or incisor proclination. Moreover, many previous studies have not tested the equation's reliability in mild spacing and crowding cases, where precision is most critical.

The objective of this study is to evaluate the accuracy of Ramanujan's equation for predicting maxillary arch perimeter before and after orthodontic treatment in non-extraction borderline cases. By comparing mathematically derived perimeter values with those measured manually using a brass wire technique, this study aims to validate the equation as a predictive tool in clinical orthodontics.

MATERIAL AND METHODS

This retrospective observational cohort study was conducted following the strengthening of the reporting of observational studies in epidemiology guidelines. The study was carried out over 18 months and received ethical clearance from the institutional ethics committee.

This retrospective observational study was conducted on 70 maxillary orthodontic study models (35 pre-treatment and 35 post-treatment) from non-extraction cases presenting with borderline crowding and generalized spacing.

Inclusion criteria and exclusion criteria

The study included maxillary dental casts from patients exhibiting either mild crowding (<5 mm) or generalized spacing, with fully erupted permanent dentition extending from the first molar to the contralateral first molar. Only cases where the ovoid arch form was preserved post-treatment and where no extractions, distalization, or orthognathic surgery had been performed were selected. Casts were excluded if they showed evidence of interproximal reduction, extractions, surgical intervention, or if the models were broken, unclear, or incomplete. In addition, cases with dental anomalies such as peg-shaped or supernumerary teeth were also excluded from the study.

Measurement protocol:

1. Semi-major axis (a): Measured from the line connecting distobuccal cusps of the maxillary first molars to the labial surface of the maxillary central incisors using a digital Vernier caliper [Figure 1a]
2. Semi-minor axis (b): Measured as half the inter-distobuccal cusp width of maxillary first molars [Figure 1b]
3. Arch perimeter (Manual): Measured using a 24-gauge brass wire adapted along the mid-buccal surface of teeth from one molar to the contralateral molar.[10]

To minimize observer bias and ensure consistency, each measurement, namely the semi-major axis, semi-minor axis, and manual arch perimeter, was recorded twice, with a 2-week interval between sessions. Reliability tests were conducted for each type of measurement. The high values obtained confirmed excellent intra-observer agreement, indicating consistent and reproducible data collection.

Mathematical formula used:

Perimeter of an ellipse:

$$\pi \left(\mathrm{a}+\mathrm{b}\right) \{1 + (3\mathrm{h} / (10 - \sqrt{(4-3h)}\ ))\}$$

where $h = (a-b)^2/(a + b)^2$

The calculated perimeter was denoted as Pm1 for pre-treatment and Pm2 for post-treatment.

Sample size calculation

The sample size was calculated using G*Power software (version 3.1) to ensure sufficient statistical power for detecting significant differences between predicted and manually measured arch perimeter values. A priori power analysis was performed with an effect size (f) of 0.5, significance level (α) of 0.05, and a power ($1-\beta$) of 0.80. The analysis yielded a non-centrality parameter (δ) of 2.915, a critical F-value of 2.0345, and degrees of freedom (df) of 33, resulting in a minimum total sample size requirement of 34. Therefore, to fulfill this criterion, at least 17 pre-treatment and 17 post-treatment models were needed to reliably measure the semi-major (a1, a2) and semi-minor (b1, b2) axes. However, to enhance statistical robustness and compensate for potential exclusions, a total of 70 samples (35 pre-treatment and 35 post-treatment) were included in the study.

Statistical analysis

All data were statistically analyzed using IBM Statistical Package for the Social Sciences Statistics for Windows, version 20.0 (IBM Corp., Armonk, NY, USA). To assess the reliability of repeated measurements, Cronbach's alpha was calculated for the semi-major axis, semi-minor axis, and arch perimeter values. Intra-observer agreement was further evaluated using intra-class correlation coefficients (ICC). To determine the validity of Ramanujan's equation in predicting arch perimeter, paired *t*-tests were performed to compare the mathematically calculated values with those obtained manually using the brass wire method. Pearson correlation analysis and scatter plots were used to examine the strength and direction of association between predicted and measured values. A $P < 0.05$ was considered statistically significant.

RESULTS

The reliability analysis demonstrated excellent internal consistency across all repeated measurements. Cronbach's alpha for the semi-major axis, semi-minor axis, and manual arch perimeter measurements exceeded 0.99, indicating a high level of intra-observer agreement. Similarly, ICC showed excellent reliability, with values of 0.985 for pre-treatment measurements and 0.977 for post-treatment measurements.

The comparison between predicted arch perimeter values, calculated using Ramanujan's equation, and manually measured values revealed a strong correlation in both pre- and post-treatment groups. For pre-treatment models, the mean perimeter calculated by the equation was 101.66 ± 5.83 mm, while the manual method yielded 101.34 ± 5.60 mm. The difference was not statistically significant ($P = 0.058$), confirming the accuracy of the equation [Table 1]. In the post-treatment group, the mean perimeter derived from the equation was 96.65 ± 4.14 mm compared to 96.82 ± 4.11 mm from manual measurements. This difference was also statistically non-significant ($P = 0.274$) [Table 2].

The strength of agreement between manual and calculated values was further supported by Pearson correlation and scatter plot analyses. The correlation plots for both pre- and post-treatment groups showed a linear relationship with minimal dispersion, indicating a high degree of alignment between the two measurement methods [Figures 2a and b]. The ICCs also reinforced this agreement, with ICC values indicating "excellent" reliability ($P < 0.001$) [Figure 2c].

These findings validate the use of Ramanujan's equation as a reliable mathematical model for predicting maxillary arch perimeter, with predictive errors remaining consistently below 1.2% across the dataset.

Table 1: Intraclass correlation and reliability (Cronbach's alpha) for predicted versus manual maxillary arch perimeter before treatment (pre-treatment models).

Maxillary arch perimeter by Ramanujan's equation of ellipse	Maxillary arch perimeter by manual method
Intraclass correlation coefficient	0.985
Significance "*P*" value	0.001 (Highly significant)
Inference	Excellent reliability*
Reliability by Cronbach's alpha	0.993

Table 2: Intraclass correlation and reliability (Cronbach's alpha) for predicted versus manual maxillary arch perimeter after treatment (post-treatment models).

Maxillary arch perimeter by Ramanujan's equation of ellipse	Maxillary arch perimeter by manual method
Intraclass correlation coefficient	0.977
Significance "*P*" value	0.001 (Highly significant)
Inference	Excellent reliability*
Reliability by Cronbach's alpha	0.988

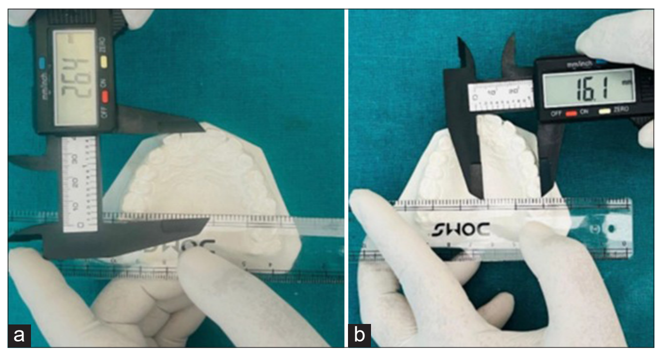

Figure 1: (a) Measurement of semimajor axis. (b) Measurement of semiminor axis.

DISCUSSION

The present study sought to evaluate the predictive accuracy and clinical reliability of Ramanujan's equation for the perimeter of an ellipse in determining maxillary arch perimeter in orthodontic patients undergoing non-extraction treatment. In borderline crowding cases where arch length-tooth material discrepancy falls near the threshold for extraction, the decision to preserve or remove teeth often hinges on small differences in available arch space.[3] Therefore, a precise, reproducible, and clinically applicable mathematical model for arch perimeter estimation can significantly assist in evidence-based treatment planning.[6] Our findings indicate that Ramanujan's equation provides a remarkably close approximation to manually measured arch perimeters,

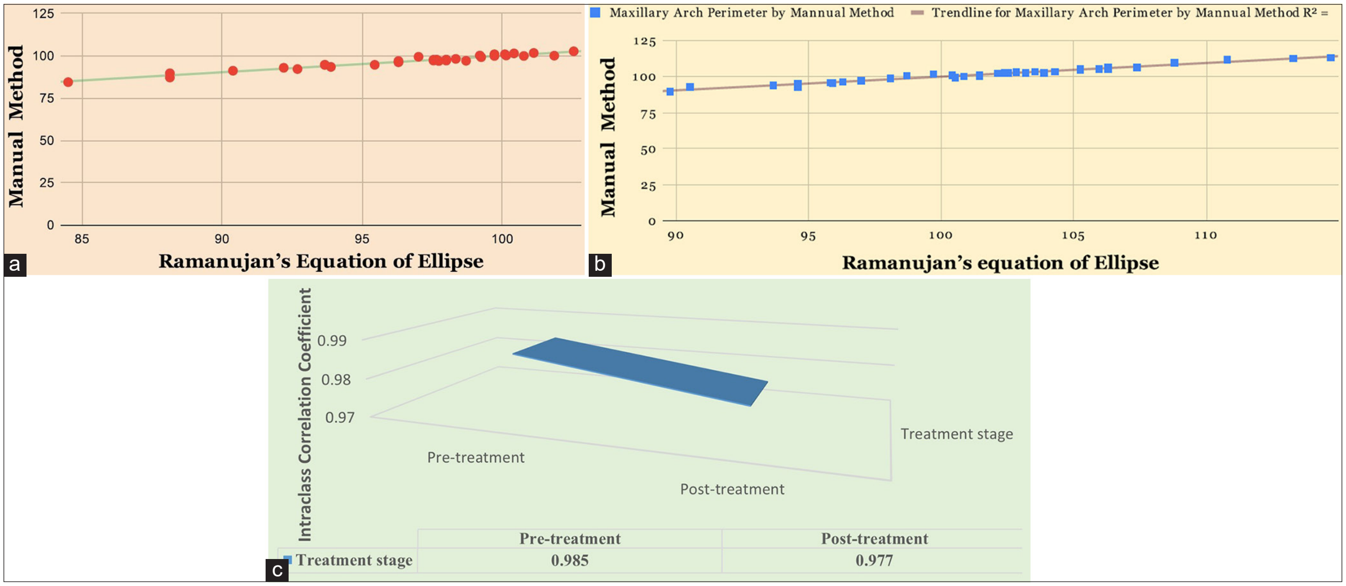

Figure 2: (a) Pretreatment scatter plot of manually measured arch perimeter versus calculated arch perimeter. (b) Post-treatment scatter plot of manually measured arch perimeter versus calculated arch perimeter. (c) Intraclass correlation coefficient between maxillary arch perimeter by Ramanujan's equation of the ellipse and by manual method pre and post-treatment.

with mean discrepancies of <1.2% in both pre- and post-treatment groups.

Measurement reliability was evaluated using both Cronbach's alpha and ICC, with all values exceeding 0.98, demonstrating excellent internal consistency and repeatability. This aligns with the methodological recommendations of Houston, who emphasized the need to minimize operator-dependent variation in orthodontic measurements.[11]

The results obtained corroborate previous work by Chung and Wolfgramm, who were among the first to apply Ramanujan's elliptical formula to the maxillary arch and reported a similar mean error rate under 1.2%. Antala *et al.* built on these findings, validating the model in a separate cohort of Indian subjects and reinforcing its cross-population applicability.[4,5] However, these earlier studies primarily focused on static arch evaluations and retroclined arches, whereas the current study also included post-treatment changes irrespective of incisor inclination, thereby demonstrating that the equation remains valid even after modifications induced by orthodontic appliances, namely molar expansion and incisor inclination correction.

From a theoretical standpoint, the dental arch has long been the subject of geometric and mathematical modeling. Various curves, including parabolas, catenaries, cubic splines, hyperbolas, and ellipses, have been explored for their fit to the natural dental arch.[6,7,12].In orthodontics, mathematical models have been used not only to describe existing arch forms but also to predict how arch perimeter may change due to various interventions using spline and Bezier functions, which showed that dental arches undergo predictable dimensional changes post-treatment.[13] However, these methods often need digital modeling and software, limiting chairside use. Ramanujan's equation, using only linear measurements, suits both digital and manual workflows, making it more clinically accessible.

Another important clinical implication is that Ramanujan's model may serve as a predictive tool during treatment planning. By calculating expected perimeter changes from proposed arch width or incisor position adjustments, clinicians can simulate non-extraction outcomes and weigh the feasibility of preserving teeth in borderline cases. This predictive approach adds objectivity to space analysis, reducing dependence on subjective interpretation or inconsistent templates. Such integration of mathematics into diagnosis echoes the recommendations of Noroozi *et al.*, who stressed the importance of combining geometric modeling with digital records for enhanced accuracy.[14,15]

This study has certain limitations that should be acknowledged. First, the analysis was restricted to ovoid-shaped maxillary arches. Other arch forms, such as square or tapering arches, exhibit different geometric characteristics that may not align well with elliptical modeling. As a result, the applicability of Ramanujan's equation to these alternate arch forms remains uncertain and warrants further investigation. Second, the study focused exclusively on the maxillary arch, omitting the mandibular arch, which may respond differently to orthodontic forces and present distinct perimeter dynamics. Including mandibular data in future research could offer a more comprehensive understanding of the equation's clinical utility.

CONCLUSION

Within the limitations of this study, Ramanujan's equation for the perimeter of an ellipse demonstrated high accuracy and reliability in predicting maxillary arch perimeter in non-extraction orthodontic cases with ovoid arch forms. The equation showed excellent agreement with manual brass wire measurements, both pre- and post-treatment, with mean errors under 1.2% and strong intra-class correlation values. These findings suggest that this mathematical model can serve as a valuable adjunct in orthodontic space analysis, particularly in borderline cases where precise estimation of arch perimeter is critical for determining treatment direction. Further research is recommended to validate its applicability across different arch forms, mandibular arches, and various malocclusions.

Ethical approval: The research/study was approved by the Institutional Review Board at Saraswati Dental College and Hospital, Lucknow, number #SA10R18042023D, dated May 16, 2023.

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